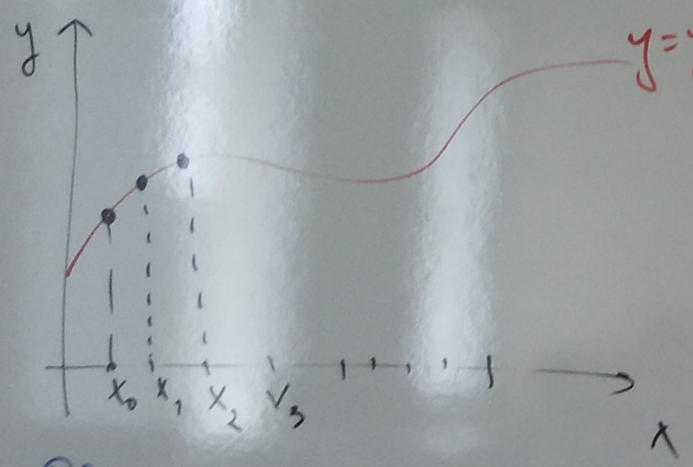
Методи на Адамс

$$\begin{cases} y' = f(x,y), & x \geq x_0 \\ y(x_0) = y_0 \end{cases}$$



x_i	y_i	$y'_i = f_i$	$\Delta y'_i = \Delta f_i$	$\Delta^2 y'_i$	$\Delta^n y'_i$
x_0	y_0	$y'_0 = f_0$	Δf_0	$\Delta^2 y'_0$	$\Delta^n y'_0$
x_1	y_1	$y'_1 = f_1$	$\Delta y'_1$	$\Delta^2 y'_1$	
⋮	⋮	⋮			
			$\Delta y'_{n-1}$	$\Delta^2 y'_{n-2}$	
x_n	y_n	$y'_n = f_n$			

Основна идея:

I. Знаем точките $y_0, y_1, \ldots, y_n$

II. Построяваме интерп. полином на Нютон за ф-ята $y'(x) = f(x,y)$

III. $y(x) = \int y'(x)\,dx$

Интерп. полином на Нютон "назад"

$$y(t) \approx P_n(t) = y_n + t\,\Delta_{n-1} + \frac{t(t+1)}{2!}\Delta^2_{n-2} + \frac{t(t+1)(t+2)}{3!}\Delta^3_{n-3} + \ldots + \frac{t(t+1)\ldots(t+n-1)}{n!}\Delta^n_0$$

$$t = \frac{x - x_n}{h}$$

$\Delta y'_k = f_k$

Методи на Адамс

II. Интерп. полином на Нютон "назад" за ф-ята $y'(x)$

① Построяваме табл. на крайни разлики $\Delta y'_i$

x_i	y_i	$y'_i = f_i$	$\Delta y'_i = \Delta f_i$	$\Delta^2 y'_i$	$\Delta^4 y'_i$
x_0	y_0	$y'_0 = f_0$	$\Delta y'_0$	$\Delta^2 y'_0$	$\Delta^4 y'_0$
x_1	y_1	$y'_1 = f_1$	$\Delta y'_1$	$\Delta^2 y'_1$	
⋮	⋮	⋮			
x_{n-1}			$\Delta y'_{n-1}$	$\Delta^2 y'_{n-2}$	
x_n	y_n	$y'_n = f_n$			

② Конструиране полинома по ф-лата → Интерп. полином на Нютон "назад"

$$y(t) \approx P_n(t) = y_n + t\,\Delta_{n-1} + \frac{t(t+1)}{2!}\Delta^2_{n-2} + \frac{t(t+1)(t+2)}{3!}\Delta_{n-3} + \ldots + \frac{t(t+1)\ldots(t+n-1)}{n!}\Delta^n_0$$

$$y'(t) = y'_n + t\,\Delta y'_{n-1} + \frac{t(t+1)}{2!}\Delta^2 y'_{n-2} + \frac{t(t+1)(t+2)}{3!}\Delta^3 y'_{n-3} + \ldots + \frac{t(t+1)\ldots(t+k-1)}{k!}\Delta^k y'_{n-k} + R_k(t)$$

k → брой точки, които използваме

$k+1$

$$t = \frac{x - x_n}{h} \Rightarrow x = h.t + x_n$$

$$t = \frac{x - x_n}{h}$$

Δy^p_k

Методи на Адамс

II. Интерп. полином на Нютон "назад" за ф-ята $y'(x)$

① Построяваме табл. на крайни разлики $\Delta y'$

② Конструиране полинома по ф-лата

$$y'(t) = y'_n + t\,\Delta y'_{n-1} + \frac{t(t+1)}{2!}\Delta^2 y'_{n-2} + \frac{t(t+1)(t+2)}{3!}\Delta^3 y'_{n-3} + \dots + \frac{t(t+1)\dots(t+k-1)}{k!}\Delta^k y'_{n-k} + R_k(t)$$

$P_n(t)$

k → брой точки, които използваме

$k+1$

III. Интегриране $y'(x)$ в границите от x_n до x_{n+1}

$$\int_{x_n}^{x_{n+1}} y'(x)\,dx = y(x)\Big|_{x_n}^{x_{n+1}} = y(x_{n+1}) - y(x_n) = y_{n+1} - y_n$$

$$\int_{x_n}^{x_{n+1}} y'(x)\,dx = h\int_0^1 P_n(t)\,dt$$

смяна на променливите:

$$\int_{x_n}^{x_{n+1}} \dots dx \longleftrightarrow \int_0^1 \dots h\,dt$$

$x = x_n + t.h$

$dx = h.dt$

x	x_n	x_{n+1}
t	0	1

$t = \frac{x - x_n}{h} \Rightarrow x = h.t + x_n$

Методи на Адамс

$$y_{n+1} - y_n = h\int_0^1 \left(y_n' + t\Delta y_{n-1}' + \frac{t(t+1)}{2!}\Delta^2 y_{n-2}' + \frac{t(t+1)(t+2)}{3!}\Delta^3 y_{n-3}' + \dots + \frac{t(t+1)\dots(t+k-1)}{k!}\Delta^k y_{n-k}' + R_k(t)\right)dt =$$

$$= h\Bigg[y_n' \int_0^1 dt + \Delta y_{n-1}' \int_0^1 t\,dt + \Delta^2 y_{n-2}' \int_0^1 \frac{t(t+1)}{2!}dt + \Delta^3 y_{n-3}' \int_0^1 \frac{t(t+1)(t+2)}{3!}dt + \dots$$

$$+ \dots + \Delta^k y_{n-k}' \int_0^1 \frac{t(t+1)\dots(t+k-1)}{k!}dt + \int_0^1 R_k(t)dt \Bigg]$$

$$\int_0^1 dt = t\Big|_0^1 = 1 - 0 = 1$$

$$\int_0^1 \frac{t(t+1)}{2!}dt = \frac{5}{12}$$

$$\int_0^1 t\,dt = \frac{t^2}{2}\Big|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

$$\int_0^1 \frac{t(t+1)(t+2)}{3!}dt = \frac{3}{8}$$

$$y_{n+1} = y_n + h\left[y_n' + \frac{1}{2}\Delta y_{n-1}' + \frac{5}{12}\Delta^2 y_{n-2}' + \frac{3}{8}\Delta^3 y_{n-3}' + \dots\dots \right]$$

Методи на Адамс

Опр. $\Delta_i^k = \Delta_{i+1}^{k-1} - \Delta_i^{k-1}$ - крайна разлика от к-ти ред (Def: $\Delta^0 f_i = f_i$)

$$\Delta y'_{n-1} = y'_n - y'_{n-1}$$

$$\Delta^2 y'_{n-2} = \Delta y'_{n-1} - \Delta y'_{n-2} = (y'_n - y'_{n-1}) - (y'_{n-1} - y'_{n-2}) = y'_n - 2y'_{n-1} + y'_{n-2}$$

$$\Delta^3 y'_{n-3} = \Delta^2 y'_{n-2} - \Delta^2 y'_{n-3} = (y'_n - 2y'_{n-1} + y'_{n-2}) - (y'_{n-1} - 2y'_{n-2} + y'_{n-3}) =$$

$$= y'_n - 3y'_{n-1} + 3y'_{n-2} - y'_{n-3}$$

x_i	y_i	$f_i := y_i'$
		⋮
		y'_{n-1}
	y_n	y'_n

<u>Екстраполационни формули на Адамс:</u>

$y_{n+1} = y_n + h \cdot y'_n$ - едностъпков Адамс $O(h^2)$

$y_{n+1} = y_n + h\left[\frac{3}{2}y'_n - \frac{1}{2}y'_{n-1}\right]$ - двустъпков $O(h^3)$

$y_{n+1} = y_n + \frac{h}{12}\left[23y'_n - 16y'_{n-1} + 5y'_{n-2}\right]$ - тристъпков $O(h^4)$

$\frac{3}{2} + \frac{5}{12} = \frac{23}{12}$

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$$y_{n+1} = y_n + h\left[y'_n + \frac{1}{2}\Delta y'_{n-1} + \frac{5}{12}\Delta^2 y'_{n-2} + \frac{3}{8}\Delta^3 y'_{n-3} + \dots\dots\right\}$$