Influence of Josephson current second harmonic on stability of magnetic flux in long junctions

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Abstract. We study the long Josephson junction (LJJ) model which takes into account the second harmonic of the Fourier expansion of Josephson current. The dependence of the static magnetic flux distributions on parameters of the model are investigated numerically. Stability of the static solutions is checked by the sign of the smallest eigenvalue of the associated Sturm-Liouville problem. New solutions which do not exist in the traditional model, have been found. Investigation of the influence of second harmonic on the stability of magnetic flux distributions for main solutions is performed.

1. Motivation and model

The physical properties of the magnetic flux in Josephson junctions (JJ) are the contemporary superconducting electronics base. For the traditional JJ the current-phase dependence is a sinusoidal function. Such a model is described by the sine-Gordon equation. For a sufficiently wide class of JJ the superconducting Josephson current as a function of magnetic flux φ can be represented as a sine series [1, 2]:

$$I_S = I_c \sin\varphi + \sum_{m=2}^{\infty} I_m \sin m\varphi\,. \tag{1}$$

It was recognized recently that the higher harmonics in this expansion are important in many applications, in particular, in junctions like SNINS and SFIFS, where S is a superconductor, I is an insulator, N is a normal metal and F is a weak metallic ferromagnet [1, 3]. The interesting properties of LJJ with an arbitrarily strong amplitude of second harmonic in current phase relation were considered in [4].

Using only first two terms of the expansion (1) leads to the double sine-Gordon equation (2SG) [2].

$$-\varphi'' + a_1 \sin\varphi + a_2 \sin 2\varphi - \gamma = 0\,, x \in (-l; l)\,. \tag{2}$$

Here and below the prime means a derivative with respect to the coordinate x. The magnitude γ is the external current, l is the semilength of the junction, a_1 and a_2 are the normalized amplitudes of the first and second harmonics of the Josephson current [1, 5]. All the magnitudes are dimensionless.

The boundary conditions for (2) have the form

$$\varphi'(\pm l) = h_e, \tag{3}$$

where h_e is external magnetic field.

Numerical solution of the nonlinear boundary problem (2), (3) is solved on the basis of the continuous analog of Newton's method [7].

Stability and bifurcations of static solutions $\varphi(x, p)$, where $p = (l, a_1, a_2, h_e, \gamma)$ are analyzed on the basis of numerical solution of the corresponding Sturm-Liouville problem [6]:

$$-\psi'' + q(x)\psi = \lambda\psi, \quad \psi'(\pm l) = 0, \quad q(x) = a_1 \cos\varphi + 2a_2 \cos 2\varphi. \tag{4}$$

The minimal eigenvalue $\lambda_0(p) > 0$ corresponds to the stable solution. In case $\lambda_0(p) < 0$ solution $\varphi(x, p)$ is unstable. The case $\lambda_0(p) = 0$ indicates the bifurcation with respect to one of the parameters p. We characterize the solutions of equation (2) by number of fluxons $N(p)$ which is defined as

$$N(p) = \frac{1}{2l\pi} \int_{-l}^{l} \varphi(x)\, dx\,. \tag{5}$$

2. Deformation of the Meissner solution M_0

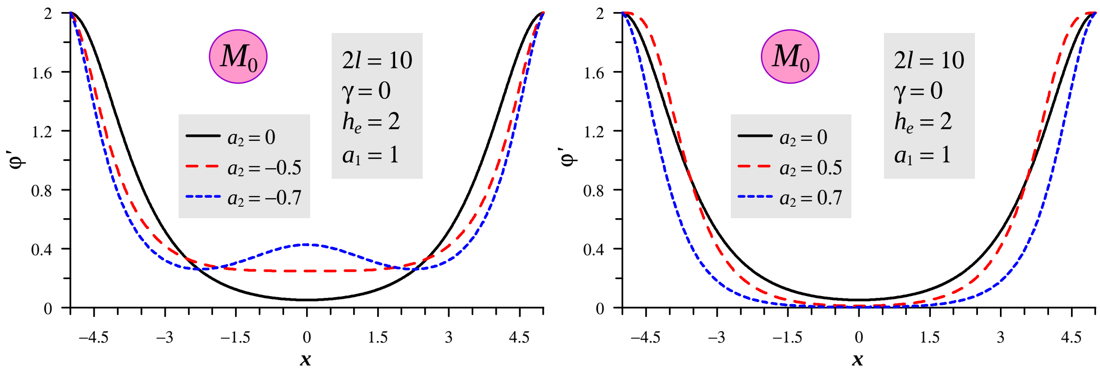

Figure 1. The internal magnetic field of the Meissner solution M_0 for LJJ with $2l = 10$, $\gamma = 0$, $h_e = 2$ and $a_1 = 1$ at different values of the parameter a_2. Left figure shows φ' at negative a_2, right one shows at positive a_2.

In the "traditional" case $a_2 = 0$ two trivial solutions $\varphi = 0$ and $\varphi = \pi$ of (2), (3) are known at $\gamma = 0$ and $h_e = 0$, which are denoted by M_0 ($N[M_0] = 0$) and M_π ($N[M_\pi] = 1$), respectively. Accounting of the second harmonic $a_2 \sin 2\varphi$ leads to the appearing of two additional solutions $\varphi = \pm\arccos(-a_1/2a_2)$ denoted as $M_{\pm ac}$ ($N[M_{\pm ac}]$ are not integer numbers and depend on the value of second harmonic). Stability properties of trivial solutions in dependence on parameter a_2 are considered in [7] and [8].

All solutions with $N[\varphi] = 0$ we denote here by M_0, even they are changed by the influence of the external magnetic field and parameter a_2.Within the LJJ model, the basic Meissner solution M_0 demonstrates the screening of the external magnetic field. Deformation of M_0 at different values of parameter a_2 under the external magnetic field $h_e = 2$ is shown in Fig. 1. At $a_2 < 0$ (left panel) the screening effect diminishes when the coefficient a_2 decreases and it amplifies at $a_2 > 0$ (right panel) with growing of a_2.

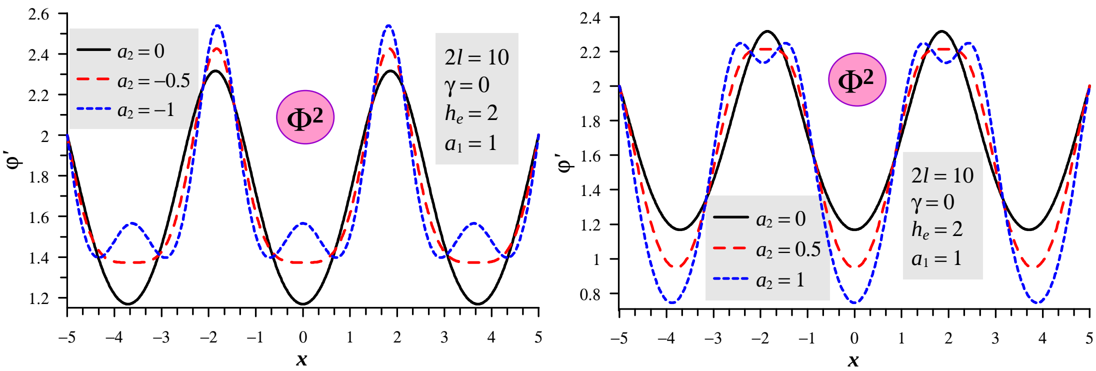

Figure 2. The internal magnetic field of the distribution Φ^2 at $2l = 10$, $\gamma = 0$, $h_e = 2$ and $a_1 = 1$ at different values of the parameter a_2. Left figure shows φ' at negative a_2, right one shows at positive a_2.

3. Deformation of the fluxon solutions Φ^n

The solutions with $N[\varphi] = 1$ which are not $\varphi = \pi$ at $h_e = 0$ and $\gamma = 0$ we denote as Φ^1. The deformation of the $\varphi'(x)$ of fluxon states under influence of the parameter $a_2 \in [-1; 1]$ was considered in [8]. We observe a qualitative change in the local minima with decrease of a_2 in the interval [-1;0] and in the local maxima with increase of a_2 in the interval [0;1]. With change of the coefficient a_2 the number of fluxons corresponding to the distribution Φ^1 is conserved [6] i.e. $\partial N/\partial a_2 = 0$ and $N[\Phi^1] = 1$. As external magnetic field h_e is growing, more complicated stable fluxon with $n = N[\varphi] = 2, 3, \ldots$ appear which are denoted by Φ^n.

The effect of the second harmonic contribution on Φ^1 was studied in [4] and [9]. It was shown in [4] that taking into account at $h_e = 0$ the second harmonics in current phase relation with $a_2 < -0.5$ leads to the appearance of the "small" fluxon state additionally to the traditional "large" one. Taking into account (5), we call the solution at $a_2 = -0.7$ in Fig. 1, left panel as M_0 (in [4] it is a "small" fluxon), because $N[M_0] = 0$. Number of fluxons of "large" fluxon is equal to $N[large] = 1$ and we denote it by Φ^1. In [9], stability properties of "large" fluxon Φ^1 at nonzero h_e have been studied. Two coexisting stable Φ^1 like fluxons were demonstrated in some region of magnetic field. Here, we investigate this effect in case of two-fluxon and three-fluxon distributions. The relation between the "small", "large" fluxons and trivial solutions under the influence of the external magnetic field and the second harmonic are the point of our further research.

In Fig. 2 the internal magnetic field of the two-fluxon distribution Φ^2 for $2l = 10$, $\gamma = 0$, $h_e = 2$ and $a_1 = 1$ at different values of the parameter a_2 is presented. We observe a strong deformation of the fluxon distributions with decrease of the parameter a_2 in the interval [-1;0]. At $a_2 = -0.5$ the curve of internal magnetic field $\varphi'(x)$ has a plateau at the points $x \approx -4$, $x \approx 0$ and $x \approx 4$ (Fig. 2, left panel). Further increase of the absolute value of a_2 leads to the transformation of these plateaus to local maxima of the internal magnetic field. Thus, accounting of the a_2 contribution qualitatively changes a shape of the fluxon distribution Φ^2. A similar deformation in the local maxima regions (points $x \approx -2$ and $x \approx 2$) is observed for $a_2 > 0$ (Fig. 2, right panel). We stress that the number of fluxons is conserved $N[\Phi^2] = 2$ for all values a_2.

4. Stability analysis of the static fluxon distribution Φ^n

The two-fluxon solution Φ^2 in LJJ with $2l = 10$ and $\gamma = 0$ is stable at $0.5 \lesssim h_e \lesssim 2.5$. Qualitatively, the behavior of the curves $\lambda_0(h_e)$ for this solution is similar to the case of one-fluxon solution Φ^1 [9]. The only difference is that in the Φ^2 case we don't have a region of

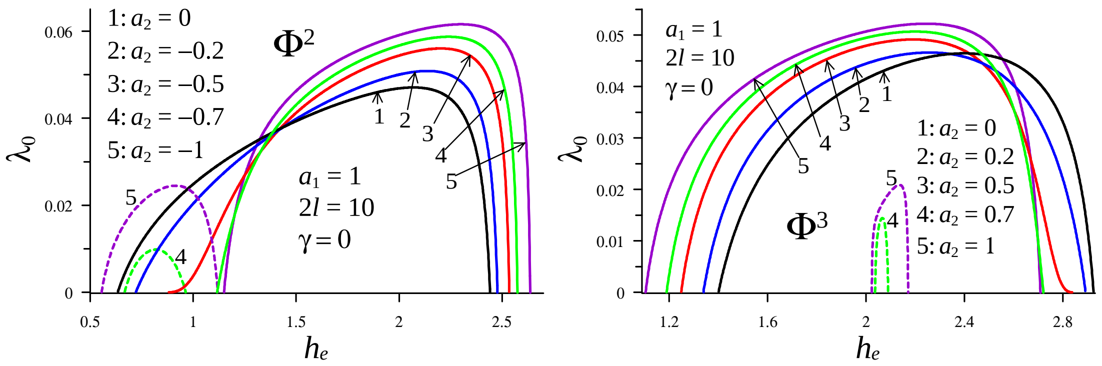

Figure 3. The dependence $\lambda_0(h_e)$ for Φ^n at $2l = 10$, $\gamma = 0$, $h_e = 2$ and $a_1 = 1$ at different values of the parameter a_2. Left figure shows $\lambda_0(h_e)$ for Φ^2 at negative a_2, right one shows $\lambda_0(h_e)$ for Φ^3 at positive a_2.

coexistence of two stable branches that was observed for negative a_2 in the Φ^1 case.

Results for $a_2 \in [-1; 0]$ are presented in Fig. 3, left panel. When a_2 decreases in $(-0.5; 0]$, the curve $\lambda_0(h_e)$ moves to the right. At $a_2 < -0.5$ the curve $\lambda_0(h_e)$ corresponding to the stable solution Φ^2 has two separate branches.

The $\lambda_0(h_e)$ curves for Φ^3 distribution for $a_2 \in [0; 1]$ are demonstrated in Fig. 3, right panel. It is seen, when a_2 is growing in the interval $[0; 1]$, the first bifurcation point moves to the left. Contrary, at $a_2 > 0.7$ the bifurcation point moves to the right. The second bifurcation point moves to the left as a_2 is growing from 0 to $a_2 \approx 0.7$. Contrary, at $a_2 > 0.7$ the second bifurcation point moves to the right. For three-fluxon state the following effect is observed. At $a_2 > 0.5$ new curves appear and we observe an existence of two different 3-fluxon states simultaneously.

For trivial solutions our calculations show the reducing of Meissner screening when the second harmonic is negative. New fluxon solutions which appear in case $a_2 \neq 0$ have a probability to be observed in the experiment. So, it would be interesting to test this.

As summary we note that our numerical investigations show that accounting of the second harmonic contributions significantly changes the shape and stability properties of trivial and fluxon static distributions in LJJ.

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